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(NASA-CR-139117) MANUAL PHOTOGRAMMETRIC ATTITUDE DETERMINATION (Aracon Geophysics Co.) 8 p

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MANUAL PHOTOGRAMMETRIC ATTITUDE DETERMINATION

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MANUAL PHOTOGRAMMETRIC ATTITUDE DETERMINATION

ARACON Geophysics is developing a system for rapid automated photogrammetric attitude determination for use with TIROS and subsequent meteorological satellites. In the event that this automated system is not completely operational at the time of launch, a temporary manual technique has been developed. This is essentially an extension of the Nadir Angle Indicator overlay used with the early TIROS vehicles, where now a series of horizons corresponding to different roll angles are drawn for a particular value of the pitch. Yaw does not affect the location of the horizon in the pictures, but can be readily obtained from the Yaw-Roll inter-relationship. Thus, in theory, a single horizon picture contains sufficient information to determine all three attitude parameters, while a sequence of pictures will suffice to determine, and locate in time the maximum roll. This in turn permits a prediction of satellite attitude for future orbits.

In practice, however, the horizon will appear at the outer edge of the pictures of TIROS IX, where distortions are the greatest. Moreover, under normal operating circumstances, the total variation of the roll and pitch angles over the entire orbit are not expected to exceed $\pm 5^{\circ}$. These factors will limit both the accuracy with which attitude may be determined for individual frames, and the ability to plot results over an orbit, thereby determining the magnitude and time of maximum roll. An estimation of anticipated accuracies will be made in a later section.

A CDC 160-A program has been written to plot within a square, representing the approximate picture edges, a series of horizons at some specified roll interval for a given value of pitch. The horizons may be plotted at one of two scales representing the 3" and the 5" effective focal lengths. The former permits a simple overlaying of the plot on the hard copy pictures ordinarily produced at the field stations but sacrifices accuracy, while the latter scale permits greater accuracy but requires projection with an enlarger of either positive or negative film onto the plotted horizons. The program to date does not use the proper lens distortion correction of TIROS IX but can readily be made to do so now that the distortion data has been made available. Input to the program includes satellite height, camera cant, pitch angle, roll interval desired, and range of roll values to be plotted. Sample horizon plots for the right-hand camera at a 4-1/8" effective focal length using the distortion corrections of another TIROS are enclosed. (The positions of the fiducials are arbitrary in these samples.)

To plot the series of horizons, a coordinate system is established at the center of the square with x directed to the right and y directed upward. The correct x coordinates are calculated with the equation below from a series of y values and the usual roll-pitch input. The number of y values used may be adjusted to alter the smoothness of the horizon plot.

$$(\gamma^2 - G^2)\, x^2 + 2(\gamma\delta)x + [\delta^2 - (y^2 + 1)G^2] = 0$$

where

$$\gamma = \cos\theta \sin R + \sin\theta \cos R \cos P$$

$$\delta = \cos\theta \cos R \cos P - \sin\theta \sin R - y \cos R \sin P$$

$$G^2 = 1 - \frac{R_e^2}{(R_e + h)^2}$$

here θ is the camera cant angle ($\theta > 0$ for left camera)

R is the roll angle

P is the pitch angle

h is the satellite height

R_e is the earth's radius

The two solutions of equation (1) represent the two horizon ends for a given y, only one of which can be seen in the focal plane. A complete derivation of equation (1) is provided in a separate section.

At the field stations, ARACON personnel using either light tables or the enlargers will determine the instantaneous roll values for individual pictures. Where using the enlargers, the operator would choose by trial and error the proper pitch curves, and after aligning the fiducials of the picture with those on the horizon plot (much as used to be done with the focus sheets), the roll is read off for that picture. The program is a short, fast one, and new overlays can be generated at each station or at some central location to replace lost or worn sheets, or if heights other than nominal should occur. As this is to be a temporary operation, plastic overlays will not be made.

The next step is to plot the roll values along the orbit and thereby determine the magnitude and location of maximum roll. Current planning gives TTCC responsibility for the determination of instantaneous roll angles from the earth-space spin ratios found from the Sanborn I-R data. These are then plotted against time to determine maximum roll using sine curve templates to aid in curve fitting. Since TTCC is already geared to plot these curves, it is suggested that the same procedure be carried out for the picture data, where now the instantaneous roll values are provided by the field stations. Should this prove impossible, ARACON has developed its own sine curve templates for curve fitting at the field stations.

As indicated previously, the horizon images will appear in the most distorted portions of the film plane. While much of this distortion can be corrected for, residual distortions of perhaps one percent will remain. Fortunately the scan lines will run normal to the horizon image, making the horizon more distinct. At the 5" effective focal length, the separation between 2° roll interval horizon curves is about 4 mm. With these considerations, it is anticipated that accuracies of $\pm$ 30' of arc for roll and $\pm 1^{\circ}$ pitch can be achieved.

DERIVATION OF HORIZON EQUATION

A cartesian coordinate system is established at the satellite with z directed upward along the vertical away from the earth, y forward along the orbital track, and x pointing to the right, perpendicular to the orbital plane.

Yaw does not affect the horizon image in the film plane.

Positive roll, R, is introduced by a positive rotation about the y axis. The new (primed) coordinate system is related to the old by the following equation:

$$\begin{aligned} x &= x' \cos R + z' \sin R \\ y &= y' \\ z &= z' \cos R - x' \sin R \end{aligned} \tag{1}$$

Positive pitch, P, is introduced by a positive rotation about the new x' axis. The pitched and rolled coordinate system (double primed) is related to the rolled system by:

$$\begin{aligned} x' &= x'' \\ y' &= y'' \cos P - z'' \sin P \\ z' &= z'' \cos P + y'' \sin P \end{aligned} \tag{2}$$

Camera cant, θ is added by a positive roll about the y'' axis to the left-hand camera. (θ is negative for the right-hand camera.)

$$\begin{aligned} x'' &= y''' \cos \theta + z''' \sin \theta \\ y'' &= y''' \\ z'' &= z''' \cos \theta - x''' \sin \theta \end{aligned} \tag{3}$$

The triple primed coordinate system is now aligned with the satellite with $-z'''$ along the left camera axis.

Consider now the bundle of rays extending from the earth's horizon to the satellite. These rays form a right circular cone with apex at the satellite. The equation of this cone in our original coordinate system is:

$$x^2 + y^2 = \tan^2 \alpha \, z^2 \tag{4}$$

where α is the nadir angle of the horizon image. In terms of satellite height, h, and earth radius, R_e,

$$\tan^2 \alpha = \frac{R_e^2}{(R_e + h)^2 - R_e^2} \tag{5}$$

By combining equations (1), (2), and (3) we can find (x, y, z) in terms of (x''', y''', z''') and substitute into equation (4) to determine the equation for the cone in the triple primed (rolled, pitched and canted) coordinate system. The intersection of the plane $z''' = -f$ with the cone is a conic section representing the appearance of the horizon in the focal plane. We now re-write equation (4) for the rolled, pitched, and canted coordinate system arbitrarily setting $z''' = -f = -1$ for scaling purposes, and dropping the triple primes:

$$\begin{aligned} &[(x \cos \theta - \sin \theta) \cos R - (\cos \theta \cos P + x \sin \theta \cos P - y \sin P) \sin R]^2 \\ &+ [y \cos P + (\cos \theta + x \sin \theta) \sin P]^2 = \\ &\tan^2 \alpha \, [(y \sin P - x \sin \theta \cos P - \cos \theta \cos P) \cos R - (x \cos \theta - \sin \theta) \sin R]^2 \end{aligned} \tag{6}$$

Equation (6) now defines the horizon curves as they appear in the focal plane. Here the x, y coordinate system has its origin at frame center with x directed to the right and y upward. As explained in the main section of this paper, the computer program solves equation (6) for x, given a series of y values and the desired roll, pitch, and cant data. To facilitate solving for x we regroup equation (6) to get it in the form

$$Ax^2 + Bx + C = 0 \tag{7}$$

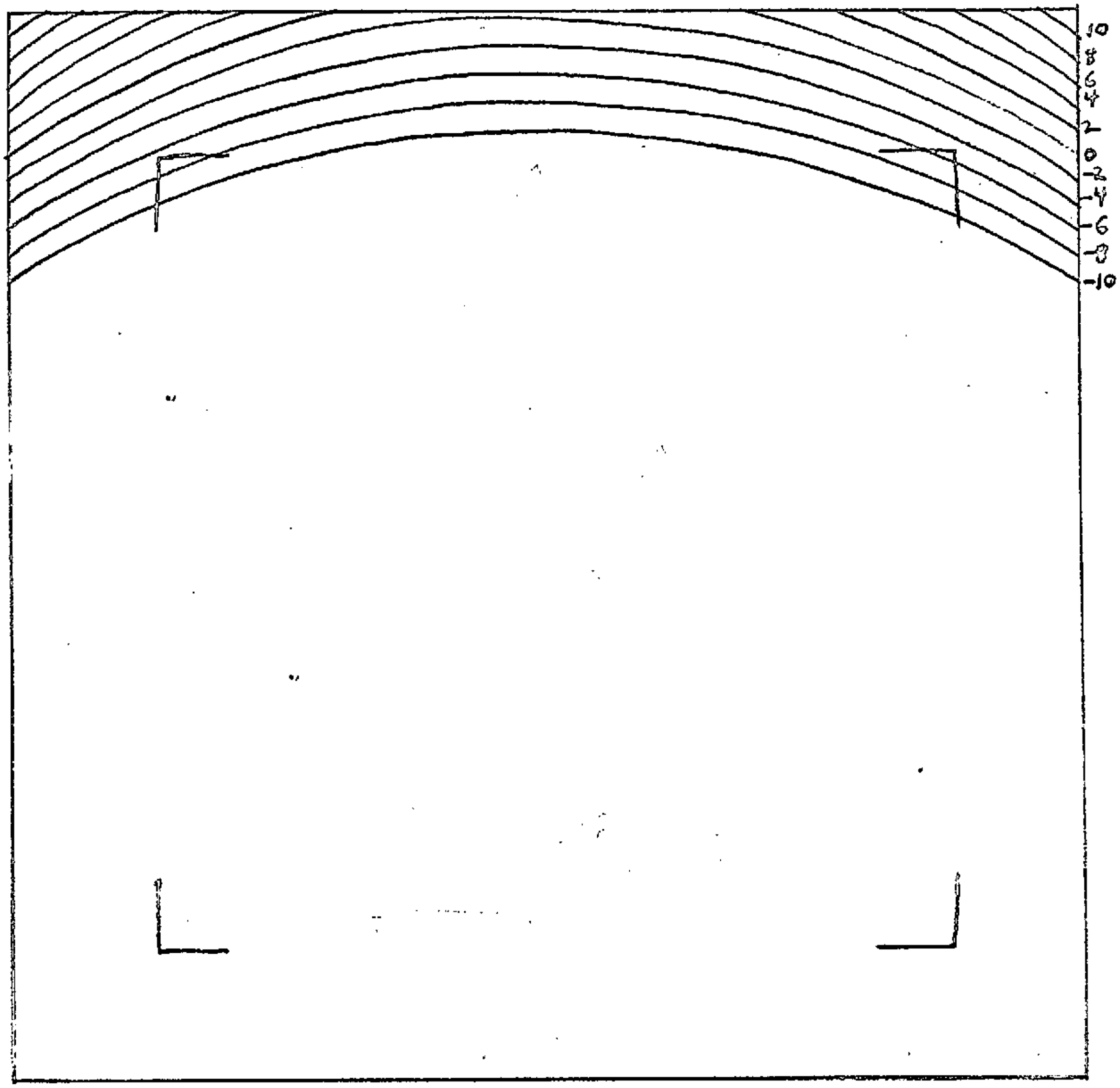
Doing this we find

$$(\gamma^2 - G^2) x^2 + 2(\gamma\delta) x + [\delta^2 - (y^2 + 1) G^2] = 0 \tag{8}$$

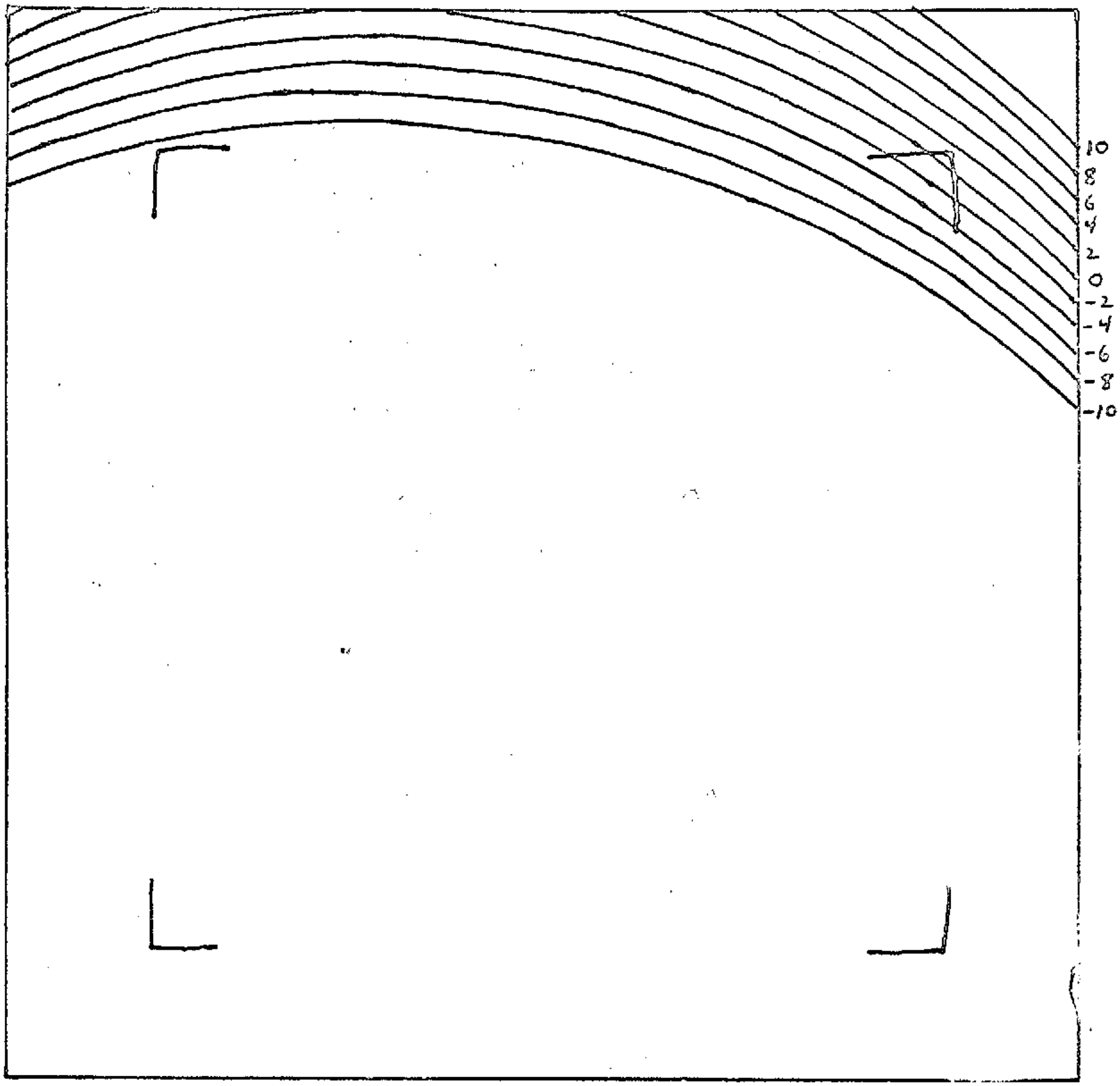
where
$$\gamma = \cos \theta \sin R + \sin \theta \cos R \cos P$$
$$\delta = \cos \theta \cos R \cos P - \sin \theta \sin R - y \cos R \sin P$$

$$G^2 = 1 - \frac{R_e^2}{(R_e + h)^2}$$

This is the equation referred to in the main section of this paper, and the one used in the actual computer programming.



Pitch .... 0°
Height .... 400 nt.mi.
Roll Interval... 2°
Roll Range .... -10° to $+10^{\circ}$



Pitch .... - 10°

Height .... 400 nt. mi.

Roll Interval ..... 2°

Roll Range .... -10° to +10°